Ordinary differential equations

1. RC circuit

1.1. Transient response

Consider a circuit made of a voltage source E, a resistor R and a capacitor C.

Kirchhoff's voltage law for this circuit gives:

$$U_{\mathrm{R}}(t) + U_{\mathrm{C}}(t) = E$$

Ohm's law for the resistor gives: $U_{\mathrm{R}}(t) = R\, i(t)$

and the charge law of the capacitor: $i(t) = \mathrm{C}\, \frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{C}}(t)$

The differential equation of Uc is therefore:

$$R\, \mathrm{C}\, \frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{C}}(t) + U_{\mathrm{C}}(t) = E$$

Outside a solve block, these relations only document the equations: they are not computed. To solve them, let's put them in a "solver" block:

$R := 500\,\Omega$ $C_1 := 1\,\mu\mathrm{F}$ $E := 12\,\mathrm{V}$ Definition of the constants

Solve block

Charging the capacitor:

$$U_{\mathrm{c}}(0\,\mathrm{s}) = 0\,\mathrm{V}$$

$$R\, C_1\, \frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{c}}(t) + U_{\mathrm{c}}(t) = E$$

$$U_{\mathrm{c}} := \mathrm{odesolve}(t, 0.01\,\mathrm{s})$$

Solve block

Discharging the capacitor:

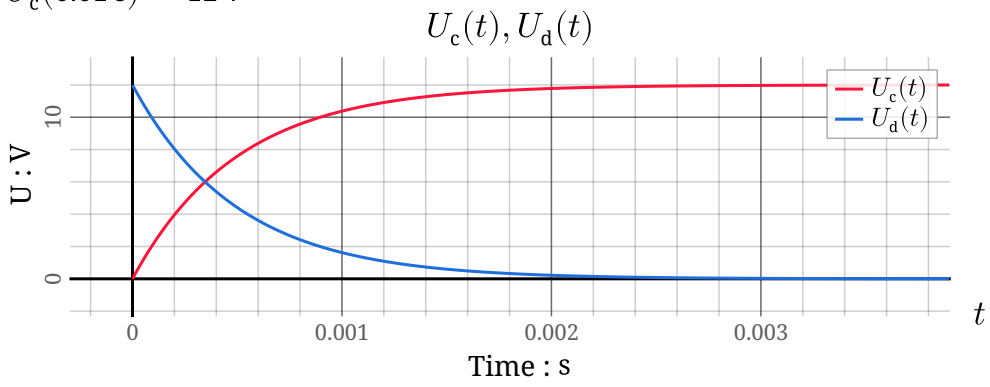
$$U_{\mathrm{d}}(0\,\mathrm{s}) = 12\,\mathrm{V}$$

$$R\, C_1\, \frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{d}}(t) + U_{\mathrm{d}}(t) = 0$$

$$U_{\mathrm{d}} := \mathrm{odesolve}(t, 0.01\,\mathrm{s})$$

$U_{\mathrm{c}}(0\,\mathrm{s}) = 0\,\mathrm{V}$

$U_{\mathrm{c}}(0.01\,\mathrm{s}) = 12\,\mathrm{V}$



1.2. Sinusoidal input

With a sinusoidal input, the applied voltage is no longer a step but a sine wave. It can be defined outside the solver block.

$$f := 150\,\text{Hz} \qquad E_\text{s}(t) := 12\,\text{V}\,\sin(2\,\pi\,f\,t)$$

Solve block

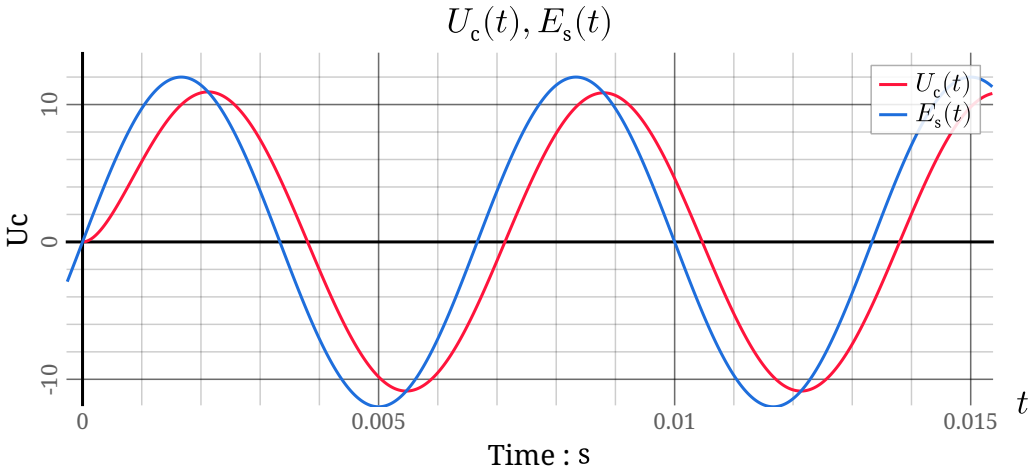
$$U_\text{c}(0\,\text{s}) = 0\,\text{V}$$

$$R\,C_1\,\frac{\text{d}}{\text{d}t}U_\text{c}(t) + U_\text{c}(t) = E_\text{s}(t)$$

$$U_\text{c} := \text{odesolve}(t, 0.05\,\text{s})$$

$$U_\text{c}(0.02\,\text{s}) = -4.627\,\text{V}$$

$$U_\text{c}(0.01\,\text{s}) = 4.627\,\text{V}$$



2. RLC circuit

To the previous circuit, made of a resistor and a capacitor, we add a coil of inductance L.

Kirchhoff's voltage law still applies:

$$U_\text{R}(t) + U_\text{C}(t) + U_\text{L}(t) = E$$

where U_L is the voltage across the coil, such that:

$$U_\text{L}(t) = \text{L}\,\frac{\text{d}}{\text{d}t}i(t)$$

The differential equation of the resulting RLC circuit is therefore:

$$\text{L}\,\text{C}\,\frac{\text{d}^2}{\text{d}t^2}U_\text{C}(t) + R\,\text{C}\,\frac{\text{d}}{\text{d}t}U_\text{C}(t) + U_\text{C}(t) = E$$

In Symbolical, it is solved in the same way as before, with a "solver" block:

$L_1 := 500\,\text{mH}$ Definition of L. The other quantities were already defined in the previous section.

Solve block

$U_c(0\,\text{s}) = 0\,\text{V}$ Zero voltage at t = 0 s

$U_c'(0\,\text{s}) = 0\,\frac{\text{V}}{\text{s}}$ Zero current at t = 0 s

$$L_1\,C_1\,U_c''(t) + R\,C_1\,U_c'(t) + U_c(t) = E$$

$U_c := \text{odesolve}(t, 0.02\,\text{s})$

