Mean power of a domestic wind turbine

An example of a problem that needs a
regression, a symbolic and a numerical analysis

1. Mechanical torque of the rotor

We start by determining the equation of the mechanical torque of a wind turbine rotor in its linear range. The torques were computed with QBlade for a wind speed of 10 m/s.

Rotor radius: $R := 0.8\,\text{m}$

Design tip speed ratio (TSR): $\lambda_0 := 5$

$M := \text{readcsv}(\text{"data_0.8m_lambda5.csv"})$

$\omega_{\text{raw}} := M_{..,0} \cdot \frac{2\,\pi}{60}$ $T_{\text{qraw}} := M_{..,1}$ RPM and torque from the CSV file

$j_{\min} := \text{argmax}(T_{\text{qraw}})$ $j_{\max} := \text{argmin}\left(\overrightarrow{|T_{\text{qraw}}|}\right)$ indices of the linear range

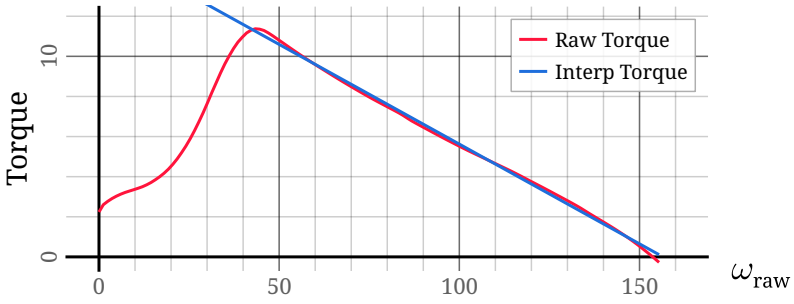
$\omega_1 := \omega_{\text{raw}_{j_{\min}..j_{\max}}}$ $T_{\text{q}} := T_{\text{qraw}_{j_{\min}..j_{\max}}}$ RPM and torque in the linear range

$\beta_{0\text{i}} := \text{intercept}(\omega_1, T_{\text{q}})$ $\beta_{1\text{i}} := -\,\text{slope}(\omega_1, T_{\text{q}})$ Parameters of the regression line

$T_{\text{qr}}(\omega) := \beta_{0\text{i}} - \beta_{1\text{i}}\,\omega$ Torque from the linear regression

Torque vs Rotation speed

$T_{\text{qraw}}, T_{\text{qr}}(\omega_{\text{raw}})$



As the parameters β0 and β1 are proportional to v^2 and v, the constants k0 and k1 follow from the interpolation:

$$k_{0\text{i}} := \frac{\beta_{0\text{i}}}{10^2} \qquad k_{1\text{i}} := \frac{\beta_{1\text{i}}}{10}$$

2. Operating point

The generator is a coreless permanent magnet model, which avoids the "sticking" effect (cogging) at start-up. Its characteristics are:

Its torque constant: $K_e \to K_e$ measured value: $K_{em} := 3.04 \ \mathrm{V}\frac{\mathrm{s}}{\mathrm{rad}}$

Its internal resistance: $R_e \to R_e$ measured value: $R_{em} := 27.8 \ \Omega$

Characteristic parameter: $\gamma := \frac{R_e}{K_e^2} \to \frac{R_e}{K_e^2}$

The torque of the generator is proportional to the current through its windings:

$$T_m = K_e\, I$$

This generator is assumed to be connected to an MPPT inverter, which injects the current into the household grid. Seen from the generator, the inverter is a load Rc, to be determined.

To determine the load seen by the generator, we consider two things:

1. the balance of the torques: the mechanical torque of the rotor is equal to the torque of the generator
2. the MPPT (Maximum Power Point Tracking) inverter does its job, finding the maximum power to export to the grid.

$$T_m := \frac{K_e^2\,\omega}{R_c + R_e} \to \frac{K_e^2\,\omega}{R_c + R_e} \quad \text{and} \quad T_h := \beta_0 - \beta_1\,\omega \to \beta_0 - \beta_1\,\omega$$

Hence the rotation speed of the rotor as a function of the load Rc:

$$\omega(R_c) := T_m = T_h \ \text{solve}, \omega \to \frac{\beta_0\,(R_c + R_e)}{K_e^2 + R_c\,\beta_1 + R_e\,\beta_1}$$

The voltage across the load is:

$$E := K_e\,\omega(R_c)\,\frac{R_c}{R_e + R_c} \to \frac{K_e\,R_c\,\beta_0}{K_e^2 + R_c\,\beta_1 + R_e\,\beta_1}$$

and the current through the load is:

$$I := \frac{E}{R_c} \to \frac{K_e\,\beta_0}{K_e^2 + R_c\,\beta_1 + R_e\,\beta_1}$$

The power dissipated in the load is therefore:

$$P(R_c) := E\,I \ \text{simplify} \to \frac{K_e^2\,R_c\,\beta_0^2}{(K_e^2 + R_c\,\beta_1 + R_e\,\beta_1)^2}$$

The optimal load is reached when:

$$R_c := \frac{\mathrm{d}}{\mathrm{d}R_c}P(R_c) = 0 \ \text{solve}, R_c \to \frac{K_e^2}{\beta_1} + R_e$$

The optimal values of the rotation speed and of the power extracted by the inverter are therefore:

$$\omega(\beta_0, \beta_1) := \omega(R_c) \text{ simplify} \rightarrow \frac{\beta_0 \left(K_e^2 + 2 R_e \beta_1\right)}{2 \beta_1 \left(K_e^2 + R_e \beta_1\right)}$$

$$P(\beta_0, \beta_1) := P(R_c) \text{ simplify} \rightarrow \frac{K_e^2 \beta_0^2}{4 \beta_1 \left(K_e^2 + R_e \beta_1\right)}$$

Replacing β0 and β1 by their relations with v, we have:

$$\beta_0 := k_0 v^2 \rightarrow k_0 v^2 \qquad \beta_1 := k_1 v \rightarrow k_1 v$$

$$\omega(v, k_0, k_1, K_e, R_e) := \omega(\beta_0, \beta_1) \rightarrow \frac{K_e^2 k_0 v + 2 R_e k_0 k_1 v^2}{2 K_e^2 k_1 + 2 R_e k_1^2 v}$$

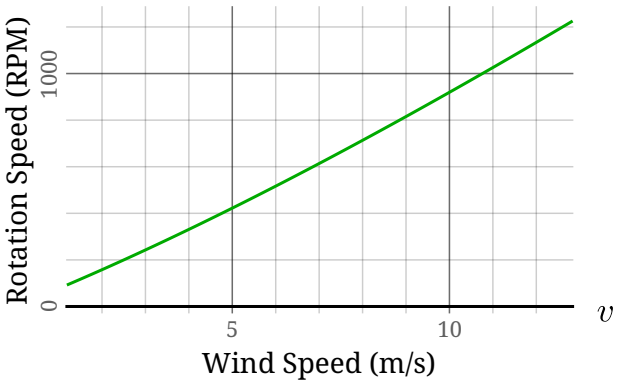
$$P(v, k_0, k_1, K_e, R_e) := P(\beta_0, \beta_1) \rightarrow \frac{K_e^2 k_0^2 v^3}{4 K_e^2 k_1 + 4 R_e k_1^2 v}$$

$$\omega(v) := \omega(v, k_{0i}, k_{1i}, K_{em}, R_{em}) \rightarrow \frac{0.086 v^2 + 1.439 v}{0.005 v + 0.184}$$

$$P(v) := P(v, k_{0i}, k_{1i}, K_{em}, R_{em}) \rightarrow \frac{0.224 v^3}{0.011 v + 0.368}$$

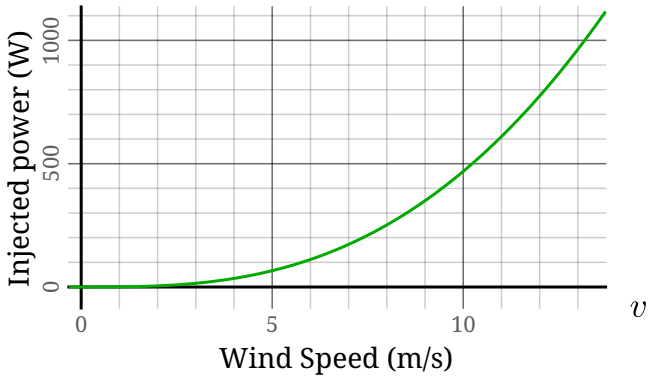
Wind Turbine Rotation speed

$$\omega(v) \cdot \frac{60}{2 \pi}$$



Wind turbine electric injected power

$$P(v)$$



3. Mean annual power

Knowing the mean wind speed at a given site, the distribution of the wind speed at this site is given by the Weibull distribution:

$$f(v, \theta, k) := \frac{k}{\theta} \left(\frac{v}{\theta}\right)^{k-1} e^{-\left(\frac{v}{\theta}\right)^k}$$

- θ is the scale parameter, related to the mean wind speed
- k is the shape parameter

In moderately hilly regions, the parameter k is close to 2, which turns the Weibull distribution into a Rayleigh distribution.

$$f(v,\theta) := f(v,\theta,2) \rightarrow \frac{2\,v\,e^{-\frac{v^2}{\theta^2}}}{\theta^2}$$

The mean wind speed is obtained by integrating the distribution over all the possible wind speeds.

$$v_{\text{mean}} := \int_0^\infty v\,f(v,\theta)\,\mathrm{d}v \text{ assume, } \theta > 0 \rightarrow \frac{\sqrt{\pi}\,\theta}{2}$$

Knowing the mean wind speed, the scale parameter θ follows:

$$\theta := v_{\text{mean}} = 4.1 \text{ solve, } \theta \rightarrow 4.626$$

Integrating the power produced by the generator over all the possible wind speeds finally gives the mean annual power of the wind turbine:

$$P_{\text{mean}}(\theta) := \int_0^\infty P(v)\,f(v,\theta)\,\mathrm{d}v$$

$$P_{\text{mean}}(\theta) = 66.575\text{W}$$